

Study of a Freely Falling Ellipse with a Variety of Aspect Ratios and Initial Angles

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Abstract: The dynamic behavior of an ellipse falling freely inside a Newtonian fluid is investigated. The immersed boundary method is used to simulate the cases at different aspect ratios and initial falling angles. Within the considered aspect ratio and falling angle, the trajectories fall into some modes including fluttering, tumbling, fluttering-tumbling, turning and steady falling motions. The cases are very sensitive to perturbations. Nevertheless, when the ellipse has the closer form to a circular cylinder, it can persist its position as initial due to a high moment of inertia. Furthermore, the initial falling angle does not affect the flow patterns significantly.

Keywords: Falling ellipse, immersed boundary method, fluid-structure interaction, nonlinear dynamics.

1. INTRODUCTION

It is a common fact that falling objects move precisely under the influence of gravity except when their motions are retarded through fluid interactions. For examples, leaves, tree seeds, and paper cards all follow complicated downward trajectories as they fall under the gravity. They flutter and tumble down in a seemingly unpredictable and complex manner. Some experimental and numerical works have been conducted to study the rich dynamical behavior of freely falling objects [1-6].

Belmonte *et al.* [1] and Mahadevan *et al.* [2] have done the experimental studies for the thin flat strips falling through a fluid in a vertical cell and the tumbling cards, respectively. Wang and Pesavento [3] studied the aerodynamics of freely falling plates for a quasi two-dimensional flow, which is a typical state for a leaf of business card falling in air. They performed the experimental and the direct numerical simulation of the two-dimensional incompressible Navier-Stokes equations for the falling objects. Andersen *et al.* [4, 5] have investigated the unsteady aerodynamics of fluttering and tumbling plates. The thickness-to-width ratio and the dimensionless moment of inertia were varied to observe the transitions between fluttering, tumbling, and steady descent of falling objects. Recently, Jin and Xu [6] have carried out the experimental and numerical simulations to study the unsteady aerodynamics of freely falling plates. The elliptical and rectangular plates were considered in their work. The plate falling velocity and trajectory were almost the same for the elliptical and rectangular plates under the experimental condition. The main dynamic differences due to the geometrical shapes were the angular velocity at which the rectangular plate rotated much slower than the elliptical plate.

From the previous studies, it should be noted that the case of freely falling objects are very sensitive to perturbations or initial conditions. Therefore, we investigate the effect of aspect ratio and initial angle of a freely falling ellipse immersed in a Newtonian fluid in the present study. According to the obtained results, we observe some interesting trajectories of a freely falling ellipse and categorize them into a mode diagram.

2. NUMERICAL METHODS

The non-dimensional continuity and momentum equations are written as follows

$$\nabla \cdot \mathbf{u} = 0 \tag{1}$$

$$\frac{\partial \mathbf{u}}{\partial t} = \nabla \cdot (\mathbf{u}\mathbf{u}) = -\nabla p + \frac{1}{\text{Re}} \nabla^2 \mathbf{u} + \mathbf{f} \quad (2)$$

We used the immersed boundary method [7] to solve the problems in this study. The three-step time-split is used to advance the flow field. First the velocity is stepped from the n^{th} time level to the first intermediate level “*” by solving the convective-diffusive terms without the pressure and virtual force for the momentum Equation (2). Subsequently, this step can be stated in the following form

$$\frac{\mathbf{u}^* - \mathbf{u}^n}{\Delta t} = \mathbf{S}^n, \quad (3)$$

where \mathbf{S} is the convective and diffusive terms of the momentum equations.

The intermediate velocity in Equation (3), in general, does not satisfy the divergence-free condition (1). At the second step we advance the first intermediate velocity by including the pressure term

$$\frac{\mathbf{u}^{**} - \mathbf{u}^*}{\Delta t} = -\nabla p^{n+1}. \quad (4)$$

By applying the divergence on both sides, Equation (4) becomes

$$\frac{\nabla \cdot \mathbf{u}^{**} - \nabla \cdot \mathbf{u}^*}{\Delta t} = -\nabla^2 p^{n+1}. \quad (5)$$

Due to conservation of mass we have

$$\nabla \cdot \mathbf{u}^{**} = 0. \quad (6)$$

Then substitution of Equation (6) to (5) gives the Poisson equation

$$\nabla^2 p^{n+1} = \frac{1}{\Delta t} \nabla \cdot \mathbf{u}^*. \quad (7)$$

Once Equation (7) is solved, we can advance the intermediate velocity in Equation (4). Furthermore, we update the velocity to the $(n+1)^{\text{th}}$ time level by imposing the virtual forces, \mathbf{f} , as follow

$$\frac{\mathbf{u}^{n+1} - \mathbf{u}^{**}}{\Delta t} = \mathbf{f}^{n+1}. \quad (8)$$

The virtual force term, \mathbf{f} , in Equation (8) represents the action of a solid upon a fluid. The force reveals the existence of a force to hold or drive a solid body when it is stationary or moving. To satisfy the no-slip condition for a solid motion, the force acting on the solid should ensure that the fluid velocity (\mathbf{u}) equal the solid velocity (\mathbf{u}_s) at the $(n+1)^{\text{th}}$ time step i.e. $\mathbf{u}^{n+1} = \mathbf{u}_s^{n+1}$. Hence, the virtual force is defined as the rate of momentum changes of solid body and proportional to the difference between the solid velocity at the $(n+1)^{\text{th}}$ time step and the fluid velocity at the n^{th} time step. The force exists on the solid body and zero elsewhere. Furthermore, it can be simply written as

$$\mathbf{f}^{n+1} = \eta \frac{\mathbf{u}^{n+1} - \mathbf{u}^n}{\Delta t} = \eta \frac{\mathbf{u}_s^{n+1} - \mathbf{u}^n}{\Delta t}, \quad (9)$$

where \mathbf{u}_s is the velocity of solid. η denotes the fraction of solid within a cell where η equal to 1 and 0 for solid and fluid cells, respectively. There is no different computational time between stationary and moving body cases due to a fixed grid arrangement. Subsequently it reduces computational cost and memory requirements, especially for moving object cases.

The motion of a solid body/particle, \mathbf{u}_s , will be tracked in the Lagrangian reference by the equations of linear momentum and transportation of a rigid body as follows

$$m_s \frac{\mathbf{v}_s^{n+1} - \mathbf{v}_s^n}{\Delta t} = -\iiint_{\Omega} \eta \rho \mathbf{f}^n dV + (m_s - m_f) \mathbf{g} \cdot \quad (10)$$

Eq. (10) can be rewritten in the differential form as

$$\frac{d(m_s \mathbf{v}_s)}{dt} = -\mathbf{F}_s + \mathbf{G}_s, \quad (11)$$

while the torque acting upon the solid, \mathbf{T}_s , and the translation of the solid can be stated as

$$\frac{d(\mathbf{I}_s \cdot \omega_s)}{dt} = \mathbf{T}_s \quad (12)$$

and

$$\frac{d\mathbf{x}_s}{dt} = \mathbf{v}_s. \quad (13)$$

When considering the rotation, the particle velocity inside the solid body moving with the translational velocity, \mathbf{v}_s , at the center of rotation and the angular velocity, ω , can be expressed as:

$$\mathbf{u}_s = \mathbf{v}_s + \omega \times \mathbf{r}, \quad (14)$$

where,

\mathbf{F}_s is the hydrodynamic force,

\mathbf{G}_s is the external force including gravitational and buoyancy force,

\mathbf{T}_s is the torque acting upon the solid particle,

\mathbf{I}_s is the inertia tensor,

ω is the angular velocity of the solid particle,

m_s is the mass of the particle,

\mathbf{v}_s is the translational velocity of the center of mass,

\mathbf{x}_s is the center position of the solid particle.

3. RESULTS AND DISCUSSION

We perform the simulations to study flow behaviors of a freely falling ellipse in a fluid domain. Since the flow is very sensitive to perturbation, the three values, 0° , 45° , and 90° , of the initial angle of attack of the falling ellipse with respect to the horizontal axis, are chosen in the simulations. The aspect ratio is changed also to investigate the effect of the thickness. The density of the ellipse and the fluid are 2.7 g/cm^3 and 1.0 g/cm^3 , respectively. The kinematic viscosity of the fluid is $0.005 \text{ cm}^2/\text{s}$. The chord of the ellipse is 0.648 cm while the thickness is varied.

The vortex shedding and the wake structures of the freely falling ellipse are shown in Figs. 1 and 2 for the fluttering and tumbling modes, respectively. The vortex sheds when the ellipse glides at a small initial angle (1a). The ellipse turns (1b) and vortices are shed from the leading and trailing edge (1c). Subsequently, the ellipse resumes gliding and the old leading edge becomes the new trailing edge by shedding the vortex (1d). For the tumbling motion, the ellipse glides by shedding vortex at the trailing edge (2a). The ellipse starts turning (2b) and sheds vortex from the old leading edge (2c). Turning is completed and the ellipse repeats to glide (2d).

When the ellipse glides with an angle of attack, it leads to two high pressure regions, one below the ellipse close to the leading edge and the other above the ellipse close to the trailing edge. These regions create a torque. The resulting torque rotates the ellipse as it begins to move broadside-on and slows down. If the rotation angle is larger than 90° , then the ellipse tumbles. Otherwise, it periodically oscillates from side to side.

Plot of the trajectory of the falling ellipse is presented in Fig 3. Within the considered range of the aspect ratio and α , the flow variations due to the ellipse falling motion can be categorized to be four modes, i.e. fluttering, tumbling, fluttering-tumbling, turning, and straight/steady falling modes. The motion of the falling ellipse tends to be fluttering when the ellipse is initially placed in horizontal position, $\alpha = 0^\circ$ The ellipse oscillates from side to side

as it descends with alternating gliding at a low initial angle and a rotational motion at the turning points. The rotational angle at the turning point becomes larger for a decreasing aspect ratio due to a smaller moment of inertia. The ellipse can not persist the fluttering motion and it tumbles finally (see the fluttering-tumbling mode).

In terms of the effect of the initial position, our result shows a good agreement with the previous study [8] A horizontally held ellipse should have fluttered to the ground as it falls. On the other hand, a vertically held ellipse should have fallen for a moment before tumbling and landing far from the point directly below the release point.

The fluttering and tumbling motion can be explained by following the observation of Maxwell [9]. When the ellipse glides with an angle of attack, $\alpha \neq 0$, it leads to two high pressure regions, one below the ellipse close to the leading edge, and the other above the ellipse close to the trailing edge. These regions create a torque. The resulting torque rotates the ellipse as it begins to move broadside-on and slows down. If the rotation angle is larger than 90° , then the ellipse tumbles. Otherwise, it periodically oscillates from side to side.

An initial angle, $\alpha = 45^\circ$ leads the ellipse tumbles immediately after being released except for the low aspect ratio case. Instead of tumbling, the ellipse travels from side to side. It should be noted that, when the ellipse falls in the fluid field, it can hold its motion as the initial at horizontal, $\alpha = 0^\circ$ and vertical, $\alpha = 90^\circ$ for a bigger aspect ratio because of a higher moment of inertia. The initial angle does not influence the motion significantly so long as the ellipse has a close form to a circular cylinder.

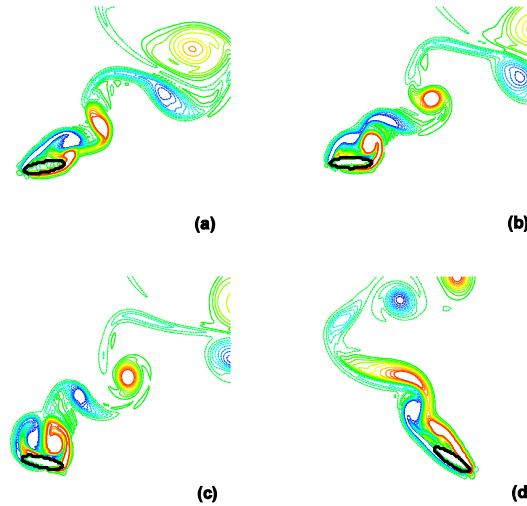


Fig. 1: The vorticity contours at the fluttering motion

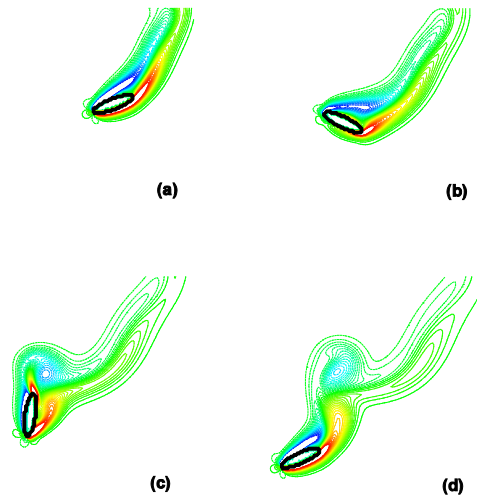


Fig. 2: The vorticity contours at the tumbling motion

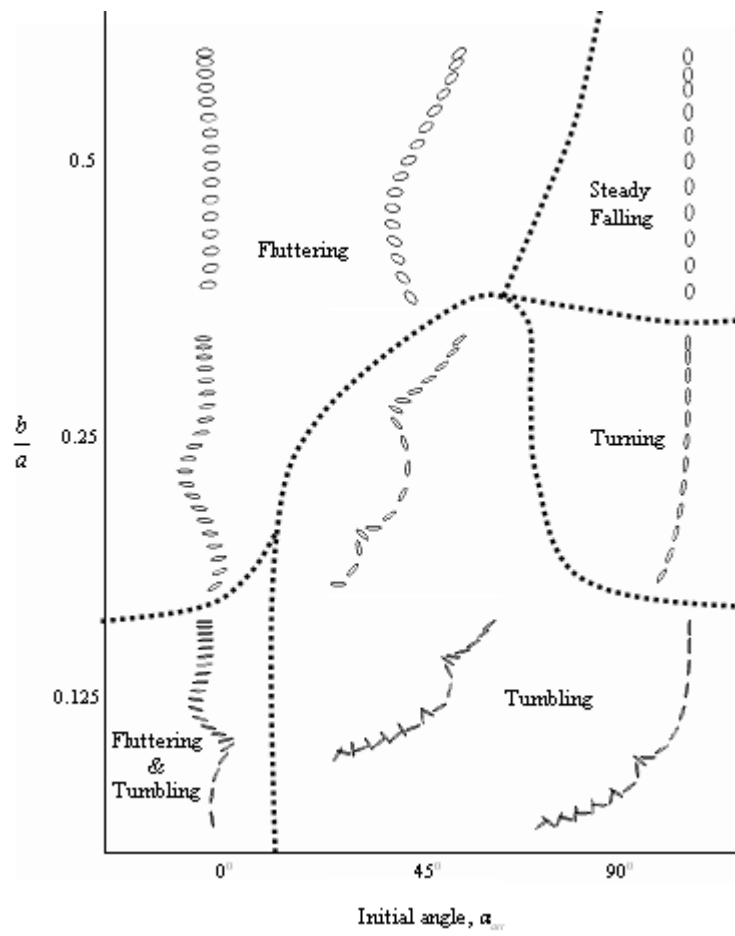


Fig. 3: Trajectory modes of a freely falling ellipse at different aspect ratio and initial angle

4. CONCLUSIONS

A freely falling ellipse with a variety of aspect ratios and initial angles has been investigated using an immersed boundary method. A horizontally held ellipse should have fluttered to the ground as it falls. On the other hand, a vertically held ellipse should have fallen for a moment before tumbling and landing far from the point directly below the release point. When the ellipse with a higher aspect ratio falls in the fluid field, it can hold its motion as initial due to a higher moment of inertia.

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